1. Problem-based instruction

Develop and implement interactive problem-based units of instruction that frame mathematics problems to help establish purposes for students to read, write, and communicate beyond being assigned or expected to do so (e.g. for their enjoyment/interest; to ask and answer abstract and authentic, disciplinary questions using mathematics, including questions about their community and individual lives; to address needs in their community or beyond; and to communicate with a specific audience).

Within these problem-based units, the teacher:

- engages students in asking mathematical questions, both practical and theoretical. (SMP1)*
- engages students in abstract and quantitative mathematical thinking and reasoning. (SMP2)
- helps students make sense of problems at different scales and persevere in solving them. (SMP1)
- helps students see the mathematics of everyday life by reading real-world scenarios incorporating or highlighting representations of mathematical problems and concepts. (SMP1, SMP2, SMP4)
- helps students imagine the theory of mathematics, or “pure mathematics,” to help students understand that mathematics can be used to wonder about the world and that such wondering can lead to applications of mathematical concepts in the world outside of school. (SMP7, SMP8)
- creates opportunities for students to enact literate mathematics identities, drawing from both within and outside of school literacy practices (e.g. having students communicate mathematical explanations to a public audience to strengthen their identities as users and doers of math) (SMP1-8)
- provides regular opportunities for students to make choices in their reading, writing, and communication about mathematics.
- offers regular opportunities for students to collaborate with peers in reading, writing, and communicating around mathematics, such as through small-group discussion of problems and opportunities to write within group projects.
- provides scaffolded support to students as needed to assist them in developing their literacy proficiencies, removing supports over time to generate more independence.
- differentiates instructional processes and product expectations to account for varying academic needs and capabilities and appropriately challenge all students.


2. Diverse texts and abundant reading opportunities in the school

The teacher:

- engages students with texts that provide entry into mathematical concepts and/or investigations of compelling problems or contexts. (SMP1)
- provides access and regular opportunities to work with:
  - a wide range of mathematical texts of varying complexities and types (i.e. print, audio, digital, and multimodal) including data representations, statistics in different formats, newspaper articles, magazines, journals, advertisements, financial information, videos, websites, diagrams, etc.
  - a wide range of texts that help students see mathematics as connected to their interests and that reflect their backgrounds, cultural experiences, possible career interests, and interactions with or uses of mathematics in everyday life.

- multiple representations and models of mathematical symbols, concepts and structures. (SMP1, SMP2, SMP7)
- engages students with digital tools and/or texts to engage in mathematical problem solving and communication. (SMP5)

notes

3. Intentional and standards-aligned instruction in disciplinary reading

The teacher:

- establishes compelling reasons for reading in mathematics (see recommendation #1 above).
- teaches students to apply disciplinary tools and concepts when working with text.
  - explicitly names, describes, and models the dispositions, strategies, and patterns of thinking typical of flexible and fluent mathematical thinkers.
  - strategically plans for which mathematical words, symbols, and phrases may need explicit definition and explanation and which are best developed through student investigation, discovery, and refinement. (SMP6)
  - for words and phrases needing explicit attention, regularly uses and explains their meanings using precise, accurate, and usable definitions.
  - for words and phrases better suited to student exploration and definition construction, provides students with supports needed to develop their own definitions through investigation, discovery, and refinement.
  - teaches students to reason abstractly and quantitatively when engaging with text-based problems. (SMP2)
  - teaches students to critically read and evaluate mathematical explanations, models, arguments, and other relevant types of mathematics texts. (SMP1, SMP3, SMP4)

- explicitly teaches the meaning, purpose, and appropriate usage of mathematical symbols (i.e., internationally recognized shorthand for complex concepts). (SMP6)
- models** how to read and make sense of word-based mathematical problems. (SMP1)
- teaches students how to look for and make use of structure when engaging with mathematical texts. (SMP7)
- teaches students how and when to look for regularity in repeated reasoning when engaging with mathematics texts. (SMP8)
- engages students in regularly translating across forms of representation (e.g., from written text to equations to tables to graphs; from words to symbols). (SMP1, SMP2)
- models for students how to write and think metacognitively through mathematical problems. (SMP1, SMP2)
- helps students read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments (SMP 3)
- engages students in authentic mathematical investigations about questions of interest to them and supports them in using mathematics to conduct those investigations. (SMP1, SMP4)

notes


3. Intentional and standards-aligned instruction in disciplinary reading (continued)

- Develops, with students, one or more questions of interest which may be answered through collecting and analyzing data.
- Develops, with students, appropriate strategies for collecting that data.
- Teaches students how to record data observations systematically and rigorously by:
  - employing multiple forms of representation (drawings, numbers, graphs, charts, word-based descriptions, etc.). (SMP1)
  - teaching students how to translate from one form to another. (SMP1, SMP2)
- Models how to discern data patterns and determine significance. (SMP5, SMP6, SMP7, SMP8)
- Models how to draw and present conclusions in oral and written language. (SMP3)
- Teaches students how to strategically use and analyze digital and online mathematics texts and tools. (SMP5)
- Scaffolds reading activities as appropriate using a range of strategies.


4. Intentional and standards-aligned instruction in disciplinary writing

The teacher:

- Establishes compelling reasons for writing and communicating about and with mathematics (see recommendation #1 above).
- Teaches students to write and communicate about and with mathematics for different authentic purposes and audiences.
- Engages students in writing to process and analyze mathematical texts and/or concepts. (SMP2)
- Teaches students to construct viable mathematical arguments and also critique the arguments of others. (SMP3)
  - Teaches students to use data and mathematical concepts, theorems, etc. to support their arguments.
- Explicitly names, describes, and models the dispositions, strategies, and patterns of thinking typical of flexible and fluent mathematical thinkers.
- Provides instruction in discipline-specific writing processes, strategies, and conventions, and attention as to why those writing norms exist in the discipline (e.g. notation conventions). (SMP3, SMP6)
- Attends to precision in mathematical language (SMP6)
- Teaches students how to write mathematical proofs by:
  - Enabling students to compare and contrast argument and mathematical proof, including their purposes. (SMP3, SMP6)
- Teaches students to construct and evaluate arguments centered on a mathematical claim and arguments. (SMP3)
- Using models of well-written proofs, contrasting them with poorly-written proofs, to help students learn the features of strong proofs. (SMP3)
- Practicing writing proofs in formats appropriate for the purpose and audience on a regular basis. (SMP3)
- Providing explicit instruction as needed in text features, writing mechanics and other standards-aligned content.
- Provides regular time for students to write, both formally and informally, aligned with instructional practice #1 above.
- Provides instruction in and opportunities for the use of technology tools to problem solve and communicate about mathematics
  - Engages students in using a diversity of tools to build mathematical models. (SMP1, SMP4, SMP5)
- Provides opportunities for students to practice using written language (e.g., proofs, models, metacognitive writing of problem solving processes) to make their conclusions public. (SMP3)
- Moves students to independent levels of research, reading, and writing in mathematics. (SMP1-8)
- Scaffolds writing activities as appropriate using a range of strategies.


5. Higher-order discussion of increasingly complex text across varying participation structures

The teacher:

- establishes compelling reasons for engaging in discussion of mathematical text (including student-produced text), representations, and/or problems (see recommendation #1 above).
- allocates time for whole-group, small-group, and pair discussion of text, and uses a range of discussion and grouping strategies.
- poses questions, and assists students in posing their own questions, that foster textual understanding and higher-order engagement with text. (SMP1)
- engages students in discussion of text types, structures, representations, and discursive practices of the discipline (e.g. precision of language, particularly with definitions). (SMP6)
- provides modeling and instruction to teach students how to generate their own higher level questions about texts.
- teaches students how to engage in productive discussions, including discussion moves appropriate to mathematics (e.g. analyzing and interpreting word problems, evaluating and applying definitions). (SMP1, SMP2, SMP6)
- provides learning activities in which students read, analyze, and discuss problems and proofs that mathematicians might use to build mathematical arguments. (SMP3)
- engages students in reasoning abstractly and quantitatively when talking about math. (SMP2)
- asks students to understand, interpret, and use mathematical symbol systems and notation in their classroom talk. (SMP2, SMP6)
- models for students how to use and connect multiple representations. (SMP1, SMP2)
- asks students to identify similar problem structures among different texts and seek connections, analogies, and patterns.
  - making connections between prior and new knowledge and represent that knowledge using mathematics. (SMP7, SMP8)
- supports students to explain or connect authentic, and/or abstract phenomena from a mathematical perspective using mathematical language. (SMP1, SMP2, SMP6, SMP7, SMP8)
- engages students in discussion around digital and media literacies and tools, and engages students in dialogue through digital tools to share and communicate ideas. (SMP5)

6. Opportunities for and instruction in speaking and listening

The teacher:

- establishes compelling reasons for presenting and listening to mathematical presentations or explanations. (SMP3, SMP4)
- teaches students to consider audience and purpose when preparing to speak or present. (SMP3, SMP4)
- provides regular opportunities for students to listen and respond to oral presentations, including those that incorporate visual and quantitative information to make students’ conclusions public (e.g., debate, reports, presentations to external audiences). (SMP3, SMP4)
- teaches students to listen to and productively critique the reasoning of others. (SMP3)
- teaches students strategies for listening and responding to mathematical explanations and/or presentations. (SMP1, SMP3, SMP6)

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7. Intentional efforts to build vocabulary, symbolic, and conceptual knowledge

The teacher:

- presents vocabulary as language in use (as opposed to presenting words in decontextualized lists).
- connects mathematical symbols to language and word meanings (SMP2, SMP6)
- attends to the need for precision in mathematical language. (SMP6)
- teaches multiple meanings or nuanced meanings of a word across different contexts and encourage students to use new words in meaningful contexts (e.g., discussion of texts, discussions of content area learning, semantic maps). (SMP6)
- provides repeated opportunities for students to review and use new vocabulary over time, including discussing ways that new vocabulary relate to one another and to students’ existing conceptual knowledge. (SMP2)
- engages students in developing their own definitions of new words through investigation, discovery, and refinement
- explicitly teaches words that build necessary knowledge for reading and writing texts of instruction and communicating key mathematics concepts. (SMP1)
- engages students in morphemic analysis (i.e., analysis of the meaning of word parts) of unfamiliar vocabulary. (SMP1)
- selects Tier 2 and Tier 3 vocabulary words to teach using disciplinary texts of instruction. (SMP1, SMP6)
- encourages talk about vocabulary among students, particularly during disciplinary learning and students’ discussions of print or digital texts. (SMP2, SMP3)
- encourages students to identify and explore new vocabulary independently and provides instruction to support this process.


8. Ongoing observation and assessment of students’ language and literacy development that informs their education

The teacher:

- engages in observation and assessment guided by:
  ❖ an understanding of language and literacy development, as well as of mathematical learning and development.
  ❖ e.g. understanding the difference between literal comprehension and inferential comprehension of any text, including mathematical texts like word problems, is helpful for teachers when developing and analyzing assessments.
- students’ strengths, areas for improvement, and socioemotional needs.
- relevant standards documents and connected mathematical practices. (SMP1-8)
  ❖ e.g. prioritizing student work as data for making instructional decisions as opposed to standardized test scores which can mask proficiencies and areas in need of development.
- administers assessments as one source of information to determine which students may need additional instructional supports.
- employs formative and diagnostic assessment tools as needed to inform specific instructional targets (e.g., assessing knowledge of specific vocabulary words taught) and engage in the instructional practices described in this document.
- provides timely and specific formative feedback to drive student learning
- involves students in the development of learning goals, as well as in supported, productive self and peer assessment / feedback. (SMP3)
- develops assessment that analyzes how students apply disciplinary tools, concepts, and literacy practices. (SMP 1-8)

notes

9. Community networking to tap into available funds of knowledge in support of developing students’ mathematical knowledge and identities

The teacher provides learning activities that:

- help students connect and build on their in-school and out-of-school literacy practices and identities.
- connect mathematics learning to family, cultural, and community issues, economic and political decisions. (SMP3, SMP4)
- address and communicate about natural and social concerns raised through community activities, issues, audiences, and forums by applying mathematical analysis and tools. (SMP3, SMP4, SMP5)
- connect to youth and popular cultural activities and concerns.
- leverage students’ literacies, learning, and knowledge to benefit their school, district, and/or community (e.g. peer education, research fairs, student to student mentoring, service learning).
- invite people representing a variety of occupations who use mathematics in their work, such as skilled tradespeople, artisans, business professionals, natural and social scientists, health professionals, and mathematicians, into the classroom (either face-to-face or via digital tools) to work with and engage in conversation with students.
- connect to and engage with math-oriented activities and spaces in local communities (financial institutions, government agencies such as labor departments, colleges and universities, laboratories).
- enable students to communicate conclusions about mathematical problems or contexts to authentic audiences. (SMP3, SMP4)


10. Metadiscursive awareness within and across academic and cultural domains (attention to language use at the "meta" level, e.g. talking about talk)

The teacher:

- supports students to connect and build on their in-school and out-of-school literacy practices and ways with words by identifying language processes and discussing how language is used based on different purposes and audiences. (SMP3, SMP6)
  - e.g. comparing how mathematicians report statistical data with how data is used in popular media.
  - e.g. calling attention to the multiple meanings of words like “evaluate” and “product” that have very specific meanings in mathematics.
  - e.g. analyzing the use of modifiers, including adjectives and adverbs, in mathematics text as compared to literary texts.
- provides learning activities that teach students to evaluate how language is used in powerful and effective ways in the discipline based on the purpose, audience, and genre of the text. (SMP2, SMP3, SMP4, SMP5, SMP6)

